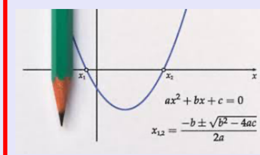
Math 125

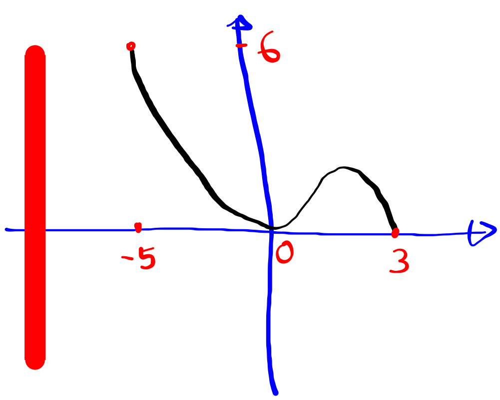
Spring 2022

Lecture 4



Review of last lecture:

Consider the graph below:



1) Domain $[-5, 3]$

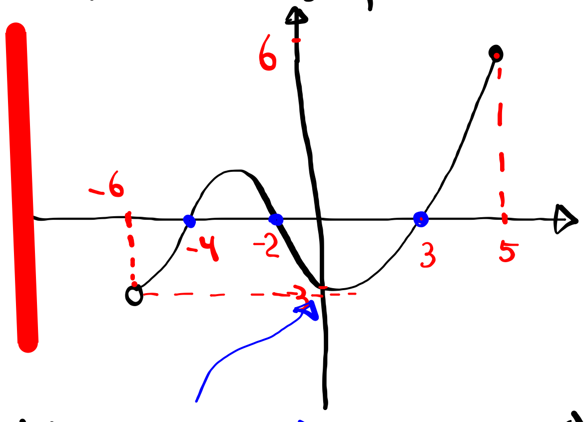
2) Range $[0, 6]$

3) Function by V.L.T.

4) Y-Int $(0,0)$

5) X-Ints $(0,0), (3,0)$

Consider the graph below



1) Domain $(-6,5]$

2) Range $[-3,6]$

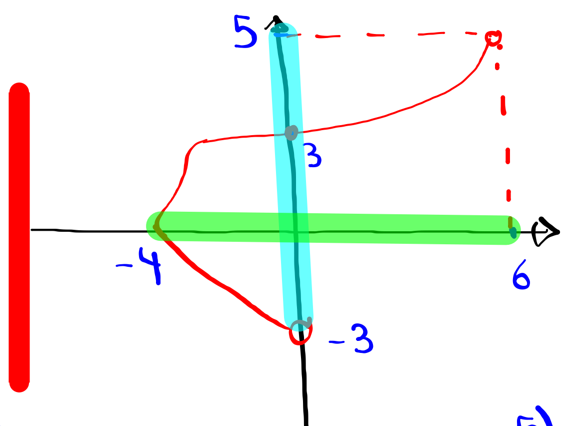
3) Function or not?

By V.L.T.

4) Y-Ints $(0,-3)$

6) X-Ints. $(-4,0)$, $(-2,0)$, $(3,0)$

Consider the graph below



1) Domain $[-4,6)$

2) Range $(-3,5)$

3) Function or not?

By V.L.T.

4) Y-Int

$(0,3)$

5) X-Int.

$(-4,0)$

$4x - 3y = 15$

1) write in linear function notation

$\underbrace{\text{linear function notation}}_{f(x) = mx + b}$ $\quad f(x) = y$

→ Isolate Y

$-3y = -4x + 15$

$y = \frac{-4}{-3}x + \frac{15}{-3}$

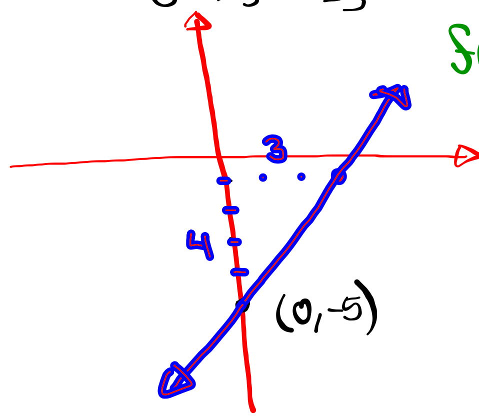
$y = \frac{4}{3}x - 5$

$f(x) = \frac{4}{3}x - 5$

2) Y-Int $(0,-5)$

3) $m = \frac{4}{3}$

4) Draw



Given $5x + 2y = 8$

1) write in linear function notation.

$2y = -5x + 8$

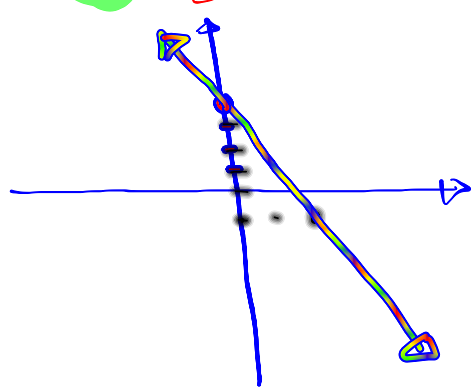
$y = \frac{-5}{2}x + \frac{8}{2}$

$f(x) = \frac{-5}{2}x + 4$

2) Y-Int $(0,4)$

3) Slope $m = \frac{-5}{2}$

4) Draw



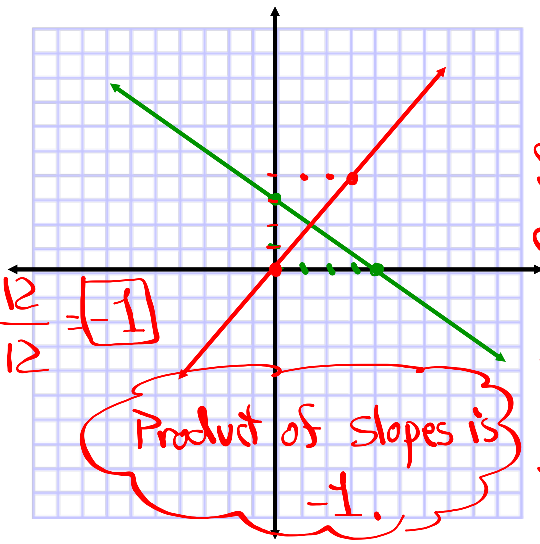
Graph $f(x) = \frac{-3}{4}x + 3$ and $g(x) = \frac{4}{3}x$ in the same coordinate system.

$m = \frac{-3}{4}$

Y-Int $(0,3)$

$m = \frac{4}{3}$

Y-Int $(0,0)$



$\frac{-3}{4} \cdot \frac{4}{3} = \frac{-12}{12} = \boxed{-1}$

Product of slopes is -1.

$f(x)$ and $g(x)$ are Perpendicular to each other

$f(x) \perp g(x)$

How to evaluate functions for a given value:

1) Replace x with given value

2) Simplify

ex: Given $f(x) = \frac{2}{3}x - 4$

Find

1) $f(0) = \frac{2}{3}(0) - 4$

$x = 0$

$= 0 - 4$

$= \boxed{-4}$

2) $f(6) = \frac{2}{\cancel{3}}(\cancel{6}^{\,2}) - 4$

$x = 6$

$= 2 \cdot 2 - 4$

$= 4 - 4$

$= \boxed{0}$

3) $f(-3) = \frac{2}{\cancel{3}}(\cancel{-3}^{\,-1}) - 4$

$x = -3$

$= 2(-1) - 4$

$= -2 - 4 = \boxed{-6}$

$f(x) = x^3 + 8$

Find

1) $f(0) = 0^3 + 8$
$= 0 + 8$
$= \boxed{8}$

2) $f(2) = 2^3 + 8$
$= 8 + 8$
$= \boxed{16}$

3) $f(-2) = (-2)^3 + 8$
$= -8 + 8$
$= \boxed{0}$

4) $f(x^2)$
$= (x^2)^3 + 8$
$= \boxed{x^6 + 8}$

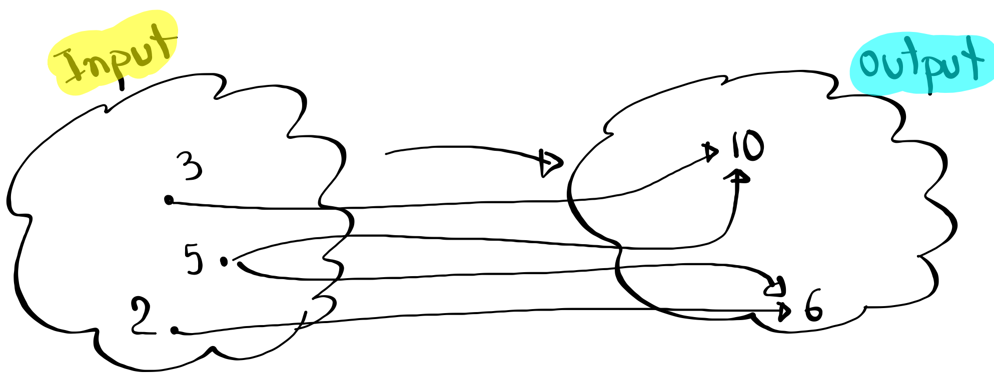
$f(x) = x^2 - 4x$

1) $f(0) = 0^2 - 4(0)$
$= 0 - 0$
$= \boxed{0}$

2) $f(4) = 4^2 - 4(4)$
$= 16 - 16$
$= \boxed{0}$

3) $f(-4) = (-4)^2 - 4(-4)$
$= 16 + 16$
$= \boxed{32}$

4) $f(x+2)$
$= (x+2)^2 - 4(x+2)$
$= (x+2)(x+2) - 4(x+2)$
$= x^2 \cancel{+2x} \cancel{+2x} + 4 \cancel{-4x} - 8$
$= \boxed{x^2 - 4}$



Domain $= \{2, 3, 5\}$ Range $= \{6, 10\}$

Function or not?

$(5, 10)$ & $(5, 6)$

Same input

Different outputs

$$f(x) = \frac{x-5}{x+4}$$

1) $f(0) = \frac{0-5}{0+4} = \boxed{\frac{-5}{4}}$

2) $f(5) = \frac{5-5}{5+4} = \frac{0}{9}$

$\frac{\text{Zero}}{\text{NonZero}} \rightarrow = \boxed{0}$

3) $f(-4) = \frac{-4-5}{-4+4} = \frac{-9}{0}$

$\boxed{\text{undefined}} \leftarrow \frac{\text{NonZero}}{\text{Zero}}$

4) $f(-5) = \frac{-5-5}{-5+4}$

$= \frac{-10}{-1} = \boxed{10}$

$f(x) = |x+5|$

Find

1) $f(0) = |0+5|$
$= |5| = \boxed{5}$

2) $f(-6) = |-6+5|$
$= |-1| = \boxed{1}$

3) $f(-5) = |-5+5|$
$= |0|$
$= \boxed{0}$

4) $f(x-5)$
$= |x \cancel{-5} \cancel{+5}|$
$= |x|$

Operations with Functions:

1) Addition:

$(f+g)(x) = f(x) + g(x)$

ex: $f(x) = 2x - 8$, $g(x) = x + 5$

Find $(f+g)(x) = f(x) + g(x)$
$= 2x - 8 + x + 5 = \boxed{3x - 3}$

2) Subtraction:

$(f-g)(x) = f(x) - g(x)$

Ex: $f(x) = x + 6$, $g(x) = 3x - 2$

Find $(f-g)(x) = f(x) - g(x)$
$= x + 6 - (3x - 2)$
$= x + 6 - 3x + 2$
$= \boxed{-2x + 8}$

3) Multiplication:

$(f \cdot g)(x) = f(x) \cdot g(x)$

Solid dot

Ex: $f(x) = 2x+3$, $g(x) = x-4$

Find $(f \cdot g)(x) = f(x) \cdot g(x) = (2x+3)(x-4)$

$= 2x^2 - 8x + 3x - 12$

$= \boxed{2x^2 - 5x - 12}$

4) Division:

$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$; $g(x) \neq 0$

ex: $f(x) = x+7$, $g(x) = x-3$

Find $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \boxed{\frac{x+7}{x-3} \ ; \ x-3 \neq 0 \ ; \ x \neq 3}$

Given $f(x) = x+6$, $g(x) = x-6$

Find

1) $(f+g)(x) = f(x) + g(x)$

$= x \cancel{+6} + x \cancel{-6} = \boxed{2x}$

2) $(f-g)(x) = f(x) - g(x)$

$= x+6-(x-6) = \cancel{x}+6-\cancel{x}+6 = \boxed{12}$

3) $(f \cdot g)(x) = f(x) \cdot g(x) = (x+6)(x-6)$

$= x^2 \cancel{-6x} \cancel{+6x} - 36$

$= \boxed{x^2 - 36}$

4) $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$

$= \boxed{\frac{x+6}{x-6} \ ; \ x-6 \neq 0 \ ; \ x \neq 6}$

x	-2	3	5
$f(x)$	3	3	-2
$g(x)$	5	-2	3

Find $(f+g)(3) = f(3) + g(3)$

$= 3 + (-2) = \boxed{1}$

Find $(f-g)(5) = f(5) - g(5)$

$= -2 - 3 = \boxed{-5}$

Find $(f \cdot g)(-2) = f(-2) \cdot g(-2) = 3 \cdot 5 = \boxed{15}$

Piece-wise Functions

Function is defined by pieces.

$$f(x) = \begin{cases} 2x & \text{if } x < 0 \\ x^2 & \text{if } x \geq 0 \end{cases}$$

Find

$f(-5) = 2(-5) = \boxed{-10}$

Find

$f(5) = 5^2 = \boxed{25}$

$f(0) = 0^2 = \boxed{0}$

$$f(x)=\begin{cases}-x^2 & \text{if } x\le 0\\[2em] \sqrt{x}+1 & \text{if } x>0\end{cases}$$

Find

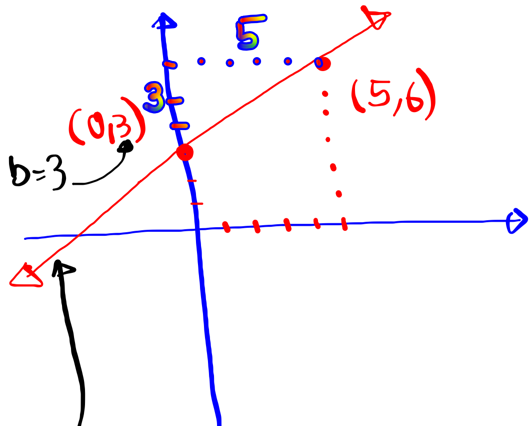
$f(-3)=-(-3)^2$
$=\boxed{-9}$

$f(0)=-(0)^2$
$=\boxed{0}$

$f(4)$
$=\sqrt{4}+1$
$=2+1$
$=\boxed{3}$

A linear function contains $(0,3)$ and $(5,6)$.

Draw this function and find its equation in linear function notation



$f(x)=mx+b$

$$\boxed{f(x)=\frac{3}{5}x+3}$$

Simplify: $(5x+3)^2 - 30x$

$= (5x+3)(5x+3) - 30x$

$= 25x^2 \cancel{+15x} \cancel{+15x} + 9 \cancel{-30x}$

$= \boxed{25x^2 + 9}$

Factor Completely: $x^2 - x - 30$

1, 30
2, 15
3, 10
6, 5

$= (x+5)(x-6)$

Zero-Factor Property:

If $A \cdot B = 0$, then $A = 0$ or $B = 0$
(maybe both)

Solve $(x-8)(x+5) = 0$

By Z.F.P.

$x - 8 = 0$ OR $x + 5 = 0$

$\boxed{x = 8}$ $\boxed{x = -5}$

Solution Set $\{-5, 8\}$

Class QZ 3

Write $3x - 5y = 10$ in Slope-Int. Form, then graph. Give slope & Y-Int. Clearly.

$3x - 5y = 10$

$-5y = -3x + 10$

$y = \frac{-3}{-5}x + \frac{10}{-5}$

$y = \frac{3}{5}x - 2$

Slope $m = \frac{3}{5}$

Y-Int $(0, -2)$